NAME _____ DATE _____ CLASS _____

Momentum and Collisions

Problem D

CONSERVATION OF MOMENTUM

PROBLEM

A 20.0 kg cannonball is fired from a 2.40×10^3 kg. If the cannon recoils with a velocity of 3.5 m/s backwards, what is the velocity of the cannonball?

SOLUTION

1. DEFINE

Given:

m_1 = mass of cannonball = 20.0 kg
m_2 = mass of cannon = 2.40×10^3 kg
$\mathbf{v_{1,i}}$ = initial velocity of cannonball = 0 m/s
$\mathbf{v_{2,i}}$ = initial velocity of cannon = 0 m/s
$\mathbf{v_{2,f}}$ = final velocity of cannon = 3.5 m/s backwards = −3.5 m/s

Unknown:

$\mathbf{v_{1,f}}$ = final mass of cannonball = ?

2. PLAN

Choose the equation(s) or situation: Because the momentum of the cannon-cannonball system is conserved and therefore remains constant, the total initial momentum of the cannon and cannonball will equal the total final momentum of the cannon and cannonball.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

Because the cannon and cannonball are initially at rest, the initial momentum for each is zero. From momentum conservation it follows that the total final momentum is also zero.

$$m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}} = 0$$

Rearrange the equation(s) and isolate the unknown(s):

$$\mathbf{v_{1,f}} = \frac{-m_2\mathbf{v_{2,f}}}{m_1}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\mathbf{v_{1,f}} = \frac{-(2.40 \times 10^3 \text{ kg})(-3.5 \text{ m/s})}{20.0 \text{ kg}} = 420 \text{ m/s}$$

$$\mathbf{v_{1,f}} = \boxed{420 \text{ m/s forward}}$$

4. EVALUATE

The velocity is positive, indicating the forward direction. The cannonball's mass is less than one-hundredth the mass of the cannon, so its speed should be over a hundred times greater than the recoil speed of the cannon.

ADDITIONAL PRACTICE

1. A student stumbles backward off a dock and lands in a small boat. The student isn't hurt, but the boat drifts away from the dock with a velocity of 0.85 m/s to the west. If the boat and student each have a mass of 68 kg, what is the student's initial horizontal velocity?

2. A coal barge with a mass of 1.36×10^4 kg drifts along a river. When it passes under a coal hopper, it is loaded with 8.4×10^3 kg of coal. What is the speed of the unloaded barge if the barge after loading has a speed of 1.3 m/s?

3. A child jumps from a moving sled with a speed of 2.2 m/s and in the direction opposite the sled's motion. The sled continues to move in the forward direction, but with a new speed of 5.5 m/s. If the child has a mass of 38 kg and the sled has a mass 68 kg, what is the initial velocity of the sled?

4. A swimmer with a mass of 58 kg and a velocity of 1.6 m/s to the north climbs onto a 142 kg raft. The combined velocity of the swimmer and raft is 0.32 m/s to the north. What is the raft's velocity before the swimmer reaches it?

5. A 50.0 g shell fired from a 3.00 kg rifle has a speed of 400.0 m/s. With what speed does the rifle recoil in the opposite direction?

6. Momentum conservation often assumes that the mass of an object remains constant throughout a process or event. However, a change in momentum can also occur when mass changes. Consider an automobile with a full tank of gasoline traveling at a velocity of 88.0 km/h to the east. The mass of the car when the fuel tank is full is 1292 kg. Suppose that the car travels along a highway that extends eastward for 600 km. By the time the car has traveled this distance, its mass is 1255 kg. What is the car's velocity at the end of the journey?

7. In 1976, Comet West was observed to break into four smaller parts as it orbited near the sun. Suppose a comet with a mass of 5.0×10^{14} kg and moving with a speed of 74.0 km/s breaks into two equal parts. One part moves 15.0° above the original orbit with a speed of 105 km/s, while a second fragment moves 30.0° below the original orbit. What is the velocity of the second comet fragment?

8. A twig floating in a small pond is initially at rest. On the twig is a snail, which begins moving along the length of the twig with a speed of 1.2 cm/s. The twig moves in the opposite direction with a speed of 0.40 cm/s. If the snail's mass is 2.5 g, what is the mass of twig?

9. A toy that is initially at rest consists of three parts that are held together by spring-loaded clips. At a given instant, the toy "explodes." Two of the pieces, which each have a mass of 25.0 g, travel with velocities of 7.0 cm/s to the south and 7.0 cm/s to the west, respectively. The third piece has a velocity of 3.3 cm/s at an angle of 45° north of east. What is the mass of the third piece?

10. An ice skater at rest catches a bag of sand moving to the north with a speed of 5.4 m/s. This causes both the skater and the bag to move to the north at a speed of 1.5 m/s. If the skater's mass is 63 kg, what is the mass of the bag of sand?