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## Momentum and Collisions

## Problem F

## KINETLG ENERGY IN PERFEGTLY INELASTIC GOLLISIONS

## PROBLEM

A ship with a mass of $4.50 \times 10^{7} \mathrm{~kg}$ and a velocity of $2.30 \mathrm{~m} / \mathrm{s}$ to the north collides with another ship whose mass is $2.30 \times 10^{7} \mathrm{~kg}$. If the speed of the second ship is $3.40 \mathrm{~m} / \mathrm{s}$ to the south, what is the change in the kinetic energy after the two ships undergo a perfectly inelastic collision?

SOLUTION
Given: $\quad m_{1}=$ mass of first ship $=4.50 \times 10^{7} \mathrm{~kg}$

$$
m_{2}=\text { mass of second ship }=2.30 \times 10^{7} \mathrm{~kg}
$$

$\mathbf{v}_{\mathbf{1}, \mathbf{i}}=$ initial velocity of first ship $=2.30 \mathrm{~m} / \mathrm{s}$ to the north

$$
=+2.30 \mathrm{~m} / \mathrm{s}
$$

$\mathbf{v}_{\mathbf{2}, \mathbf{i}}=$ initial velocity of second ship $=3.40 \mathrm{~m} / \mathrm{s}$ to the south

$$
=-3.40 \mathrm{~m} / \mathrm{s}
$$

Unknown: $\quad \mathbf{v}_{\mathbf{f}}=$ ? $\quad \Delta K E=$ ?
Use the equation for a perfectly inelastic collision to find $\mathbf{v}_{\mathbf{f}}$.

$$
\begin{aligned}
& m_{l} \mathbf{v}_{\mathbf{1}, \mathbf{i}}+m_{2} \mathbf{v}_{\mathbf{2}, \mathbf{i}}=\left(m_{1}=m_{2}\right) \mathbf{v}_{\mathbf{f}} \\
& \mathbf{v}_{\mathbf{f}}=\frac{m_{1} \mathbf{v}_{\mathbf{1}, \mathbf{i}}+m_{2} \mathbf{v}_{\mathbf{2}, \mathbf{i}}}{m_{1}+m_{2}} \\
& \mathbf{v}_{\mathbf{f}}=\frac{\left(4.50 \times 10^{7} \mathrm{~kg}\right)(2.30 \mathrm{~m} / \mathrm{s})+\left(2.30 \times 10^{7} \mathrm{~kg}\right)(-3.40 \mathrm{~m} / \mathrm{s})}{4.50 \times 10^{7} \mathrm{~kg}+2.30 \times 10^{7} \mathrm{~kg}} \\
& \mathbf{v}_{\mathbf{f}}=\frac{1.04 \times 10^{8} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}-7.82 \times 10^{7} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{6.80 \times 10^{7} \mathrm{~kg}} \\
& \mathbf{v}_{\mathbf{f}}=\frac{2.6 \times 10^{7} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{6.80 \times 10^{7} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}} \\
& \mathbf{v}_{\mathbf{f}}=0.38 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Use the equation for kinetic energy to calculate the kinetic energy of each ship before the collision and the final kinetic energy of the two ships combined.

Initial kinetic energy:

$$
\begin{aligned}
& K E_{i}=K E_{1, i}+K E_{2, i}=\frac{1}{2} m_{1} v_{1, i}^{2}+\frac{1}{2} m_{2} v_{2, i}^{2} \\
& K E_{i}=\frac{1}{2}\left(4.50 \times 10^{7} \mathrm{~kg}\right)(2.30 \mathrm{~m} / \mathrm{s})^{2}+\frac{1}{2}\left(2.30 \times 10^{7} \mathrm{~kg}\right)(-3.40 \mathrm{~m} / \mathrm{s})^{2} \\
& K E_{i}=1.19 \times 10^{8} \mathrm{~J}+1.33 \times 10^{8} \mathrm{~J}=2.52 \times 10^{8} \mathrm{~J}
\end{aligned}
$$

Final kinetic energy:

$$
\begin{aligned}
& K E_{f}=K E_{1, f}+K E_{2, f}=\frac{1}{2}\left(m_{1}+m_{2}\right) v_{f} \\
& K E_{f}=\frac{1}{2}\left(4.50 \times 10^{7} \mathrm{~kg}+2.30 \times 10^{7} \mathrm{~kg}\right)(0.38 \mathrm{~m} / \mathrm{s})^{2} \\
& K E_{f}=\frac{1}{2}\left(6.80 \times 10^{7} \mathrm{~kg}\right)(0.38 \mathrm{~m} / \mathrm{s})^{2} \\
& K E_{f}=4.9 \times 10^{7} \mathrm{~J}
\end{aligned}
$$

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Change in kinetic energy:

$$
\Delta K E=K E_{f}-K E_{i}=4.9 \times 10^{7} \mathrm{~J}-2.52 \times 10^{8} \mathrm{~J}=-2.03 \times 10^{8} \mathrm{~J}
$$

By expressing $\triangle K E$ as a negative number, $\Delta K E$ indicates that the energy has left the system to take a form other than mechanical energy.

## ADDITIONAL PRAGTIGE

1. A ball of clay with a mass of 55 g and a speed of $1.5 \mathrm{~m} / \mathrm{s}$ collides with a 55 g ball of clay that is at rest. By what percent has the kinetic energy decreased after the inelastic collision?
2. An unconventional artist creates paintings by sloshing buckets of paint onto large canvases. Suppose the canvas and easel on which it is placed have a combined mass of 4.5 kg and are initially at rest when the artist throws 1.3 kg of paint onto the canvas. The canvas, easel, and paint together slide back on the smooth floor with a speed of $0.83 \mathrm{~m} / \mathrm{s}$. What is the change in the kinetic energy after the inelastic collision?
3. The farthest source of comets is called the Oort cloud. This is a volume of space ranging from $1.5 \times 10^{10} \mathrm{~km}$ to $7.4 \times 10^{12} \mathrm{~km}$ away from the sun in which comets are loosely held by the sun's gravitational force. Suppose a comet in the Oort cloud has a mass of $1.50 \times 10^{13} \mathrm{~kg}$ and a speed of $250 \mathrm{~m} / \mathrm{s}$. This comet collides inelastically with another comet that has a mass of $6.5 \times 10^{12} \mathrm{~kg}$ and a velocity of $420 \mathrm{~m} / \mathrm{s}$ in the same direction as the first comet. What is the change in the kinetic energy of the comets after the collision?
4. Two flying fish have an inelastic collision while in mid-flight. One fish has a mass of 0.650 kg and a velocity of $15.0 \mathrm{~m} / \mathrm{s}$ to the right; the other has a mass of 0.950 kg and a velocity of $13.5 \mathrm{~m} / \mathrm{s}$ to the left. Find the change in their kinetic energy after the collision.
5. A 75.0 kg log floats downstream with a speed of $1.80 \mathrm{~m} / \mathrm{s}$. Eight frogs hop onto the $\log$ in a series of inelastic collisions. If each frog has a mass of 0.30 kg and an upstream speed of $1.3 \mathrm{~m} / \mathrm{s}$, what is the change in kinetic energy for this system?
6. What is the change in kinetic energy for the inelastic collision between the two railway cars described in problem 5 of the previous section?
7. What is the change in kinetic energy for the inelastic collision between the clay balls in problem 3 of the previous section?
8. What is the change in kinetic energy in the collision between the ball of putty and space debris described in problem 10 of Section 6E?
9. What is the change in the kinetic energy for the colliding logs in problem 4 of the previous section?
10. What is the change in kinetic energy for the inelastic collision between the four disks in problem 7 of the previous section?
