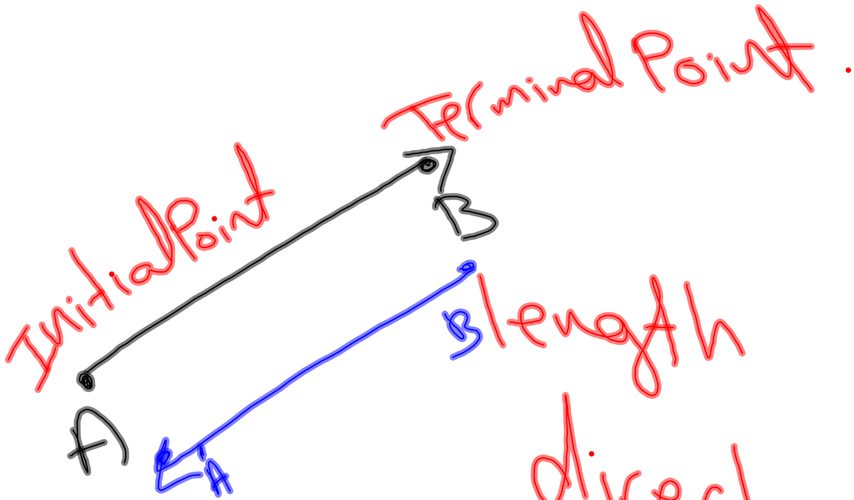
Tues, 10/2/15

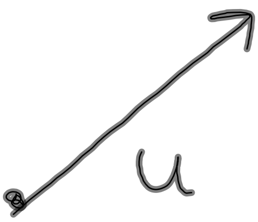
10-5 Vectors in the plane

$\overrightarrow{AB}$ <
- length
- direction.



- length
- direction.

$$\overrightarrow{AB} \neq \overrightarrow{BA}$$



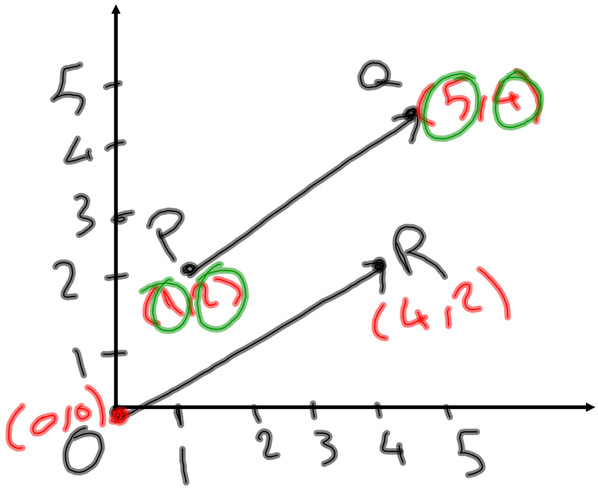
u <
- direction.
- magnitude

$\|\overrightarrow{AB}\|$

$\|u\|$

Equivalent vectors have same direction and same magnitude.

Example 1. P654



$$\|\overrightarrow{PQ}\| \sqrt{(4-2)^2+(5-1)^2} = 2\sqrt{5}$$

$$\|\overrightarrow{OR}\| = \sqrt{(4-0)^2+(2-0)^2} = 2\sqrt{5}$$

length

$$m_1 = \frac{4-2}{5-1} = \frac{1}{2}$$

direction.

$$m_2 = \frac{2-0}{4-0} = \frac{1}{2}$$

Every vector $\overrightarrow{PQ}$, such

$P=(x_1,y_1)$, $Q=(x_2,y_2)$

has an equivalent vector

with its initial Point at

the origin and its terminal

Point (x_2-x_1, y_2-y_1)

$(0,0) \qquad (a,b)$

$$\langle a, b \rangle$$

$$\langle 3, 5 \rangle$$

a, b are called the

components of the vector.

$$\|v\| = \sqrt{a^2+b^2}$$

Example 2. P655.

$$\overset{x_1}{} \quad \overset{y_1}{} \qquad\qquad \overset{x_2}{} \quad \overset{y_2}{}$$

$$P = (-2, 6), \; Q = (4, -3)$$

$$(a, b) =$$

$$(x_2 - x_1, y_2 - y_1)$$

$$(4 + 2, (-3 - 6)) = \langle 6, -9 \rangle$$

$$\|\overrightarrow{PQ}\| = \sqrt{6^2 + (-9)^2} = \sqrt{117}$$

y

magnitude.

Exercise. 1

1) Let $P = (3,3)$, $Q = (6,4)$
$O = (0,0)$ & $R = (3,1)$, as
Shown in the figure below.
Show that $\overrightarrow{PQ} = \overrightarrow{OR}$

$\cdot \ \|\overrightarrow{PQ}\| \Rightarrow \sqrt{(4-3)^2 + (6-3)^2} = \sqrt{10}$

$\|\overrightarrow{OR}\| \Rightarrow \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}$

$m\,\overrightarrow{PQ} \Rightarrow \dfrac{4-3}{6-3} = \dfrac{1}{3}$

$m\,\overrightarrow{OR} \Rightarrow \dfrac{1-0}{3-0} = \dfrac{1}{3}$

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Example 2:

Find the components & the magnitude of the vector with initial point $P=(3,-5)$ and terminal point $Q=(-6,9)$.

comp.

$$(-6-3,\ 9+5) = (-9,14)$$

$$\sqrt{(-9)^2+(14)^2} = \sqrt{277}$$

$$\langle -9, 14 \rangle$$

* Scalar multiplication.

$$V = \langle a, b \rangle$$

$$k \cdot V = \langle ka, kb \rangle$$

Const.

Example 3. P656.

$v = \langle 3, 1 \rangle$

a) $3v = \langle 9, 3 \rangle$

b) $-2v = \langle -6, -2 \rangle$

The magnitude of vector

$kv = |k| \cdot \|v\|$.

Vector addition and subtraction.

$$v = \langle a, b \rangle, u = \langle c, d \rangle$$

$$v + u = \langle a + c, b + d \rangle$$

$$v - u = \langle a - c, b - d \rangle$$

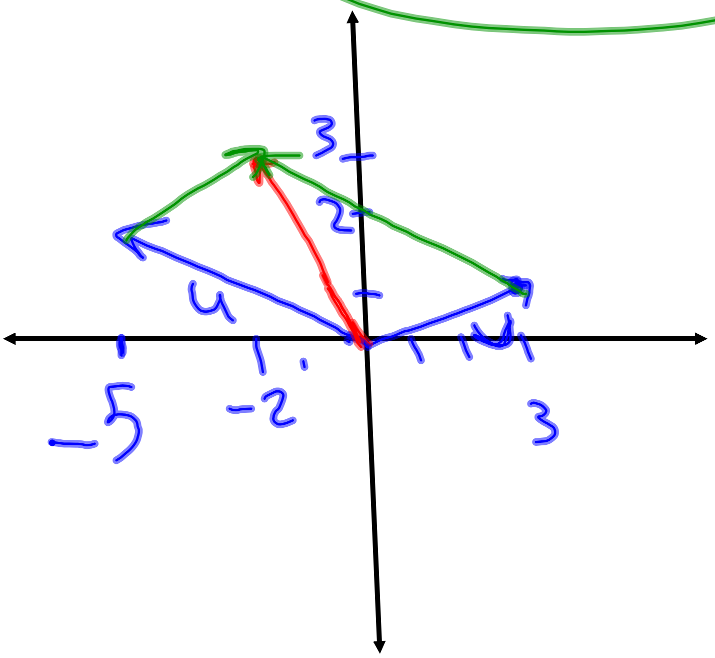
Example 4. P657.

$u = \langle -5, 2 \rangle$, $v = \langle 3, 1 \rangle$.

Find the components of $u+v$

$u+v = -5+3,\ 2+1$

$\langle -2, 3 \rangle$



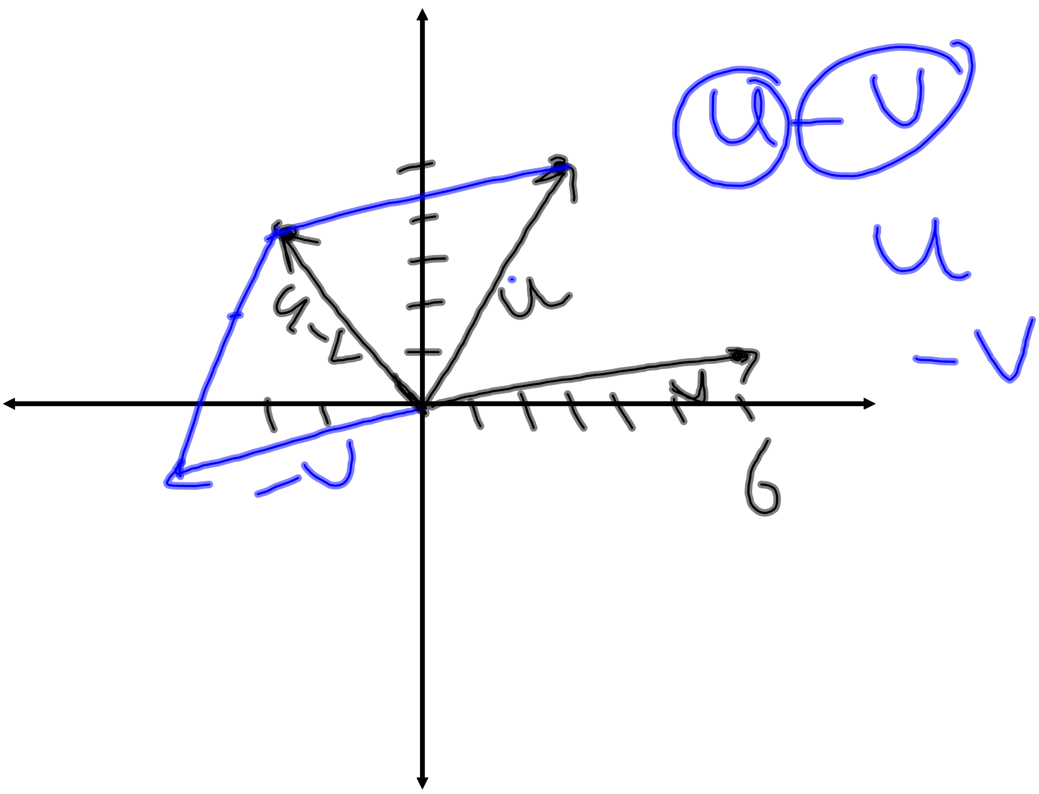
Example 5. P 658.

$u = \langle 2, 5 \rangle, v = \langle 6, 1 \rangle$

Find the components of $u - v$.

$\langle 2-6, 5-1 \rangle$

$\langle -4, 4 \rangle$



Exercise

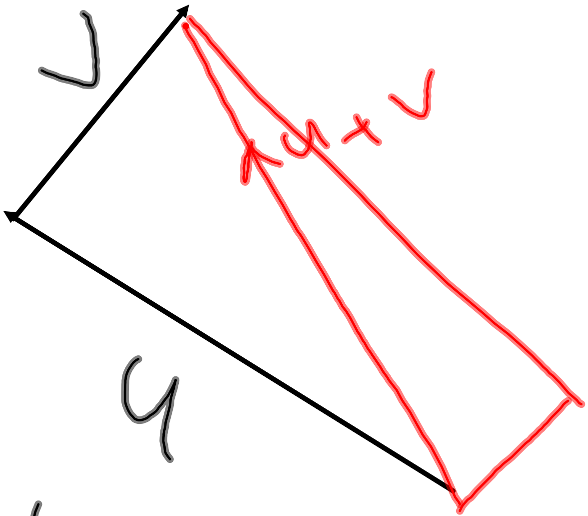
$u = \langle 5, 1 \rangle, v = \langle 7, 4 \rangle$

Find the components of

$u + v$.

Hw

P660 3, 4, 8, 9, 10, 11, 12



u+v

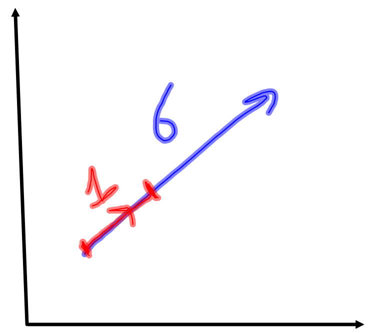
10.6 Applications on Vectors in the Plane

Unit vector. is a vector with length of 1 unit

Example 1. P661

$V = \langle 5, 12 \rangle$

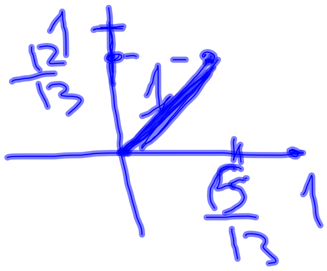
$u =$



$u = \frac{1}{6} \cdot 6 = 1$

* Components of unit vector

$= \frac{V}{\|V\|}$



$V = \langle 5, 12 \rangle$

$\|V\| \sqrt{5^2 + 12^2} = 13$

a b

$u = \frac{\langle 5, 12 \rangle}{13} = \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$

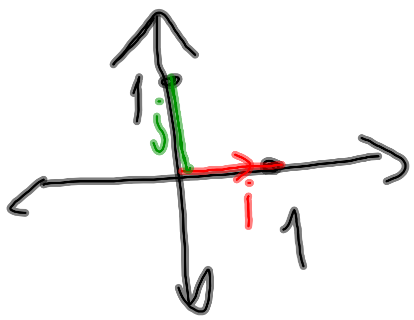
$\|u\| = \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = 1$

Exercise.

Find a unit vector u, with the same direction as the vector $V = \langle 20, 21 \rangle$

Linear Combination.

$i = \langle 1,0 \rangle, \; j \langle 0,1 \rangle$



$\langle 5,7 \rangle$

$5\overset{i}{\cancel{\langle 1,0 \rangle}}, \; 7\overset{j}{\cancel{\langle 0,1 \rangle}}$

$= 5i + 7j$

Example 2. P662.

$u = 2i - 6j$, $V = -5i + 2j$

$3u - 2V =$

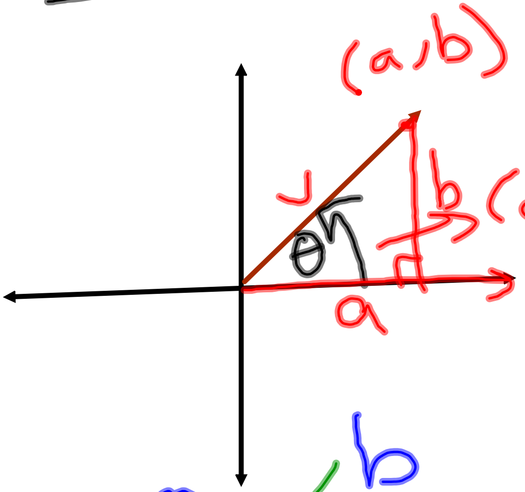
$3(2i - 6j) - 2(-5i + 2j)$

$6i - 18j + 10i - 4j$

$16i - 22j$

$\langle 16, -22 \rangle$

Direction angle



counter clockwise

$$\boxed{\tan\theta = \frac{b}{a}}$$

$$\frac{\sin\theta}{1} = \frac{b}{\|v\|}, \quad \frac{\cos\theta}{1} = \frac{a}{\|v\|}$$

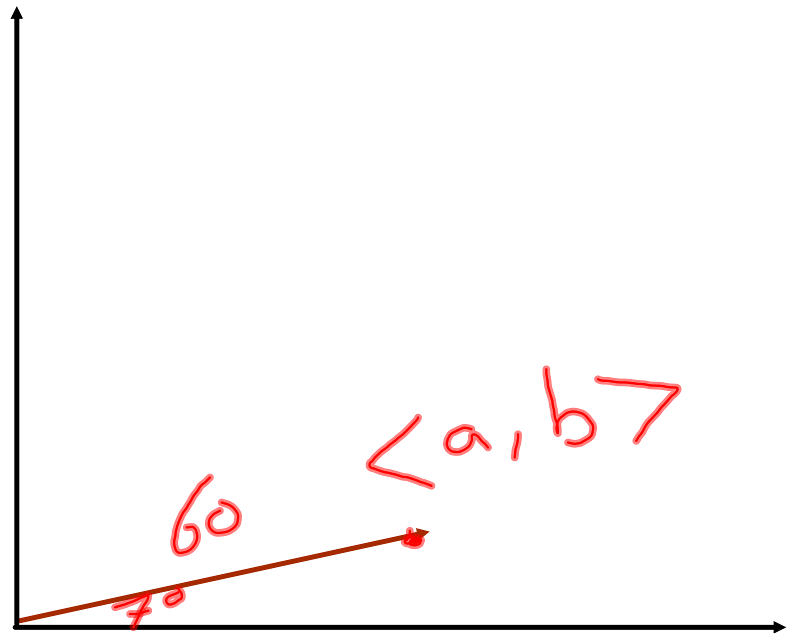
$$b = \|v\|\sin\theta, \quad a = \|v\|\cos\theta$$

$$\langle a, b \rangle$$

$$\langle \|v\|\cos\theta, \|v\|\sin\theta \rangle$$

Direction angle is the +ve angle in Standard Position with initial side is the +ve Part of x-axis and its terminal side is the given vector.

Example 3. P663.



$$\langle \|v\| \cos\theta, \|v\| \sin\theta \rangle$$

$$\langle (60 \cdot \cos 7)i, (60 \cdot \sin 7)j \rangle$$

$$\langle 59.5, 7.31 \rangle$$

or

$$59.5i + 7.31j$$

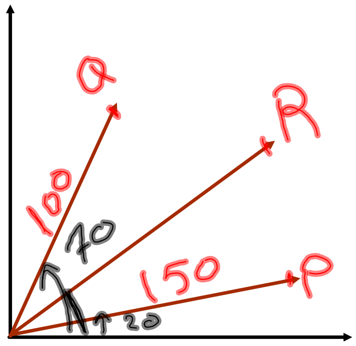
Exercise 10.6 P667

12

14

Resultant force is the sum of all forces acting on an object.

Example 5. P664.



$$\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR} \rightarrow \text{res. force.}$$

$$\langle a, b\rangle \quad \langle c, d\rangle \quad \langle a+c, b+d\rangle$$

$$\overrightarrow{OQ} = \langle 100\cos 70, 100\sin 70\rangle = \langle 34.2, 93.96\rangle$$

$$\overrightarrow{OP} = \langle 150\cos 20, 150\sin 20\rangle = \langle 140.9, 51.3\rangle$$

$$\overrightarrow{OR} = \langle 175.1, 145.26\rangle$$

$$\|\overrightarrow{OR}\| = \sqrt{175.1^2 + 145.26^2} = 227.5$$

$$\tan\theta = \frac{b}{a} = \frac{145.26}{175.1}$$

$$\theta = 39.67^\circ$$

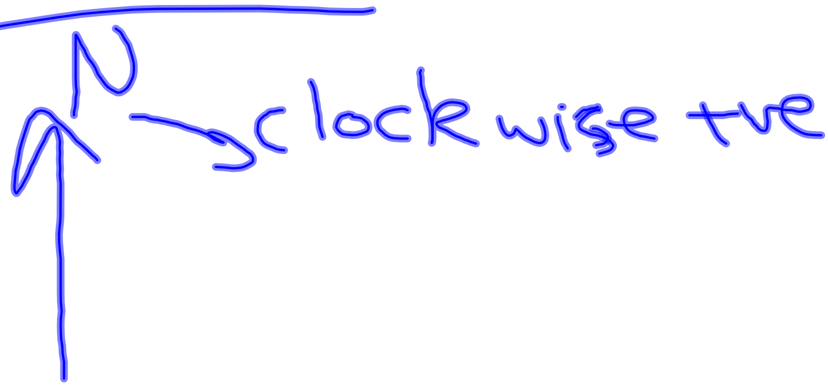
Tues, 17/2/15

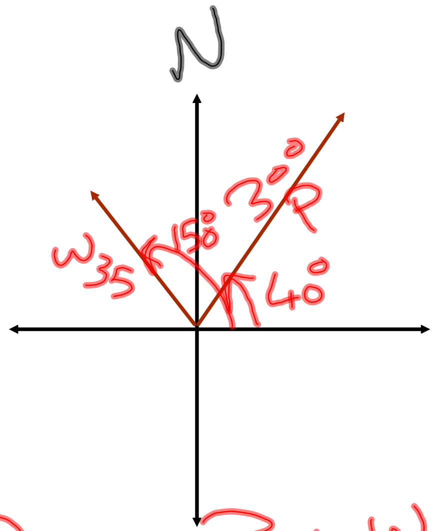
10.6 Applications on Vectors in the Plane

Example 6. P665

Example 7. P666.

Bearing





$\theta \to P = 40°$

$\theta \to w = 150°$

$$R = P + w$$

$$\langle a, b\rangle + \langle c, d\rangle$$

$$\langle \|u\| \cos\theta, \|u\| \sin\theta \rangle$$

$$P = \langle 300\cos 40, 300\sin 40\rangle$$

$$+ \qquad +$$

$$w = \langle 35\cos 150, 35\sin 150\rangle$$

$$\overset{R}{P+w} = \langle \overset{a}{199.5}, \overset{b}{210.34}\rangle$$

$$\tan\theta = \frac{b}{a} = \frac{210.34}{199.5}$$

$$\theta = 46.5°$$

$$\|P+w\| = \sqrt{199.5^2 + 210.34^2}$$

$$= 289.9$$

Exercise 10-6 P668.

33

42

H.w

P667, 668 no 10, 11, 18, 19, 21, 28, 31, 34, 36, 43

Mon, 23/2/15

10.6 A The Dot Product

If $V = \langle a, b \rangle$, $U = \langle c, d \rangle$

Then the dot product of the 2 vectors:

$$U \cdot V = ac + bd$$

Example 1. P670

a) $u = \langle \underset{a}{5}, \underset{b}{3} \rangle, v = \langle \underset{c}{-2}, \underset{d}{6} \rangle$

$u \cdot v = 5 \cdot -2 + 3 \times 6$

$= -10 + 18 = \boxed{8}$

b) $u = 4i - 2j, v = 3i - j$

$\langle 4, -2 \rangle, \langle 3, -1 \rangle$

$12 + 2 = 14$

c) $u = \langle 2, -4 \rangle, v = \langle 6, 3 \rangle$

$12 - 12 = 0$

Properties of dot Product

1] $u \cdot u = \|u\|^2$

2] $u \cdot v = v \cdot u$

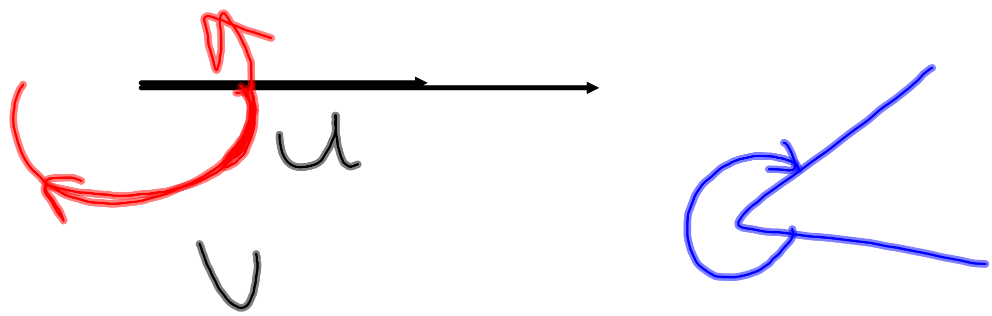
3] $u \cdot (v + w) = uv + uw$

4] $ku \cdot v = k(u \cdot v) = u \cdot kv$

5] $0 \cdot u = 0$

* Angles between vectors.

- The angle betwee u and v is the smallest angle θ formed by these 2 vectors
- The direction is ignored ((clockwise, counter clockwise)
The angle from u to v is as same as the angle from v to u
- The measure of this angle θ Lies in the interval $[0, \pi]$



$[0, 180]$ $180 < \theta < 360$

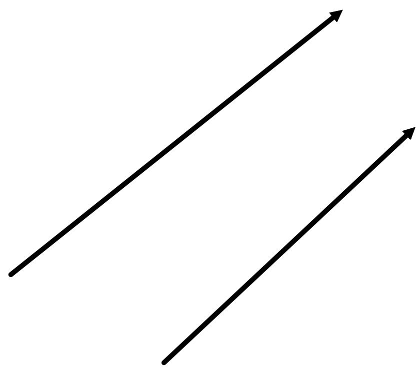
Zero Vector $= \langle 0, 0 \rangle$

Parallel vectors

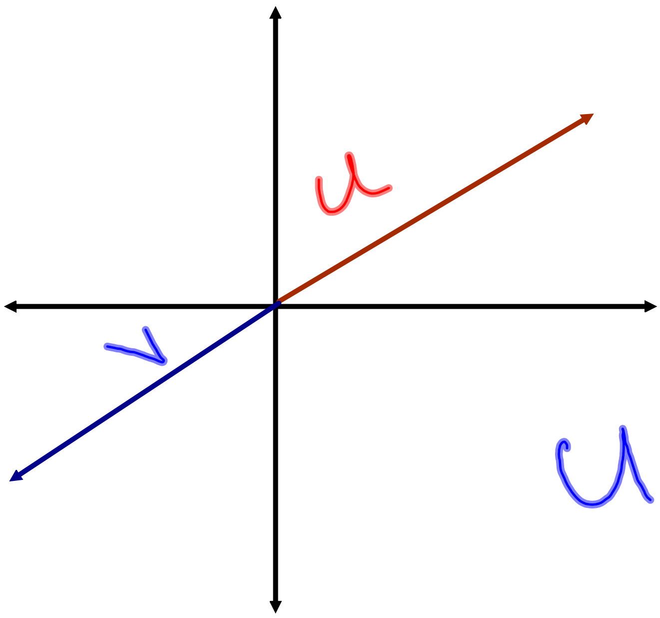
* u and v are Parallel if $u = kv$ or $v = ku$

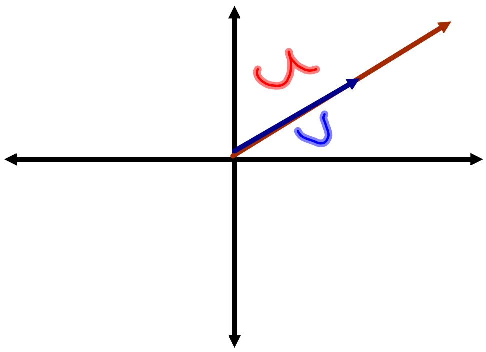
* If u and v are Parelle l, the angle θ between them is either 0 or 180

* If u and v are Parallel, then they both lie on the same St. line through the origin.

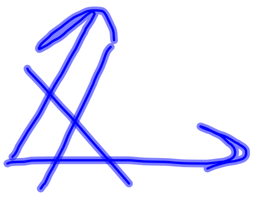


$\langle a, b \rangle$





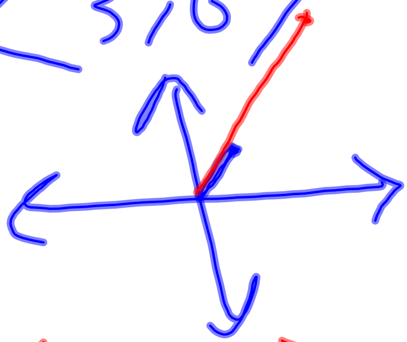
$u \parallel v$



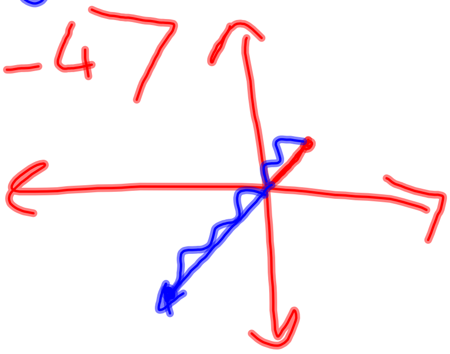
$-ku$

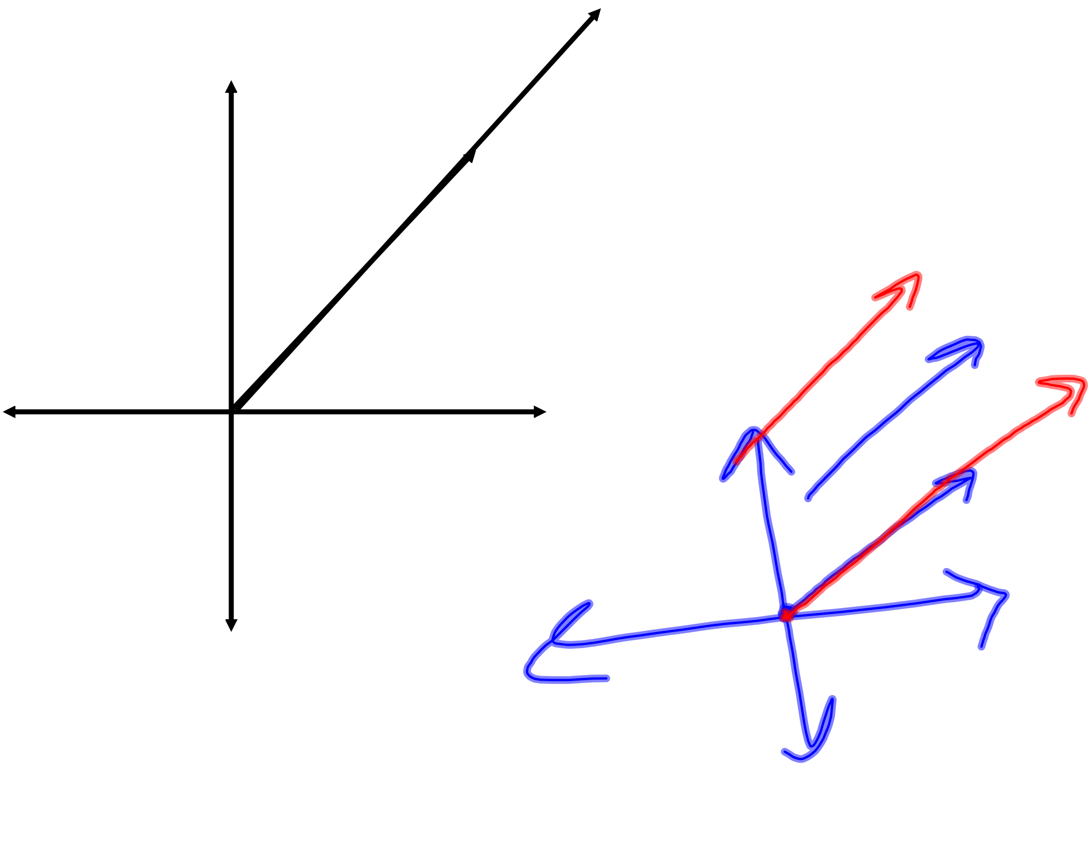
ku

$3\langle 1,2\rangle = \langle 3,6\rangle$



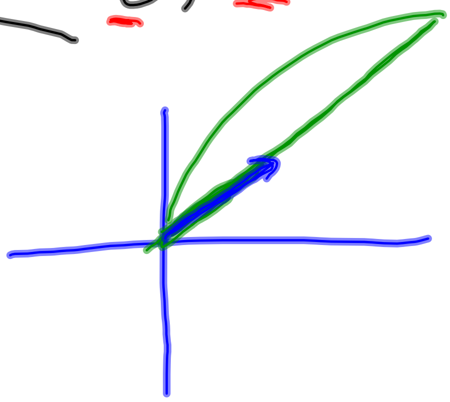
$-2\langle 1,2\rangle = \langle -2,-4\rangle$





Example 2. PG 71

$u = \langle 2, 3 \rangle, \; v = \langle 8, 12 \rangle$



$ku = v$

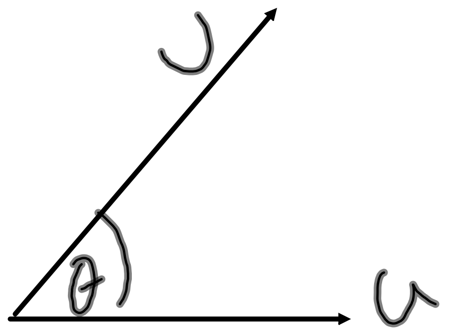
$4u = v$

They are Parallel.

Exercise

Determine whether $u = \langle 5, 4 \rangle$, $v = \langle 15, 12 \rangle$ are Parallel

Angle theorem.

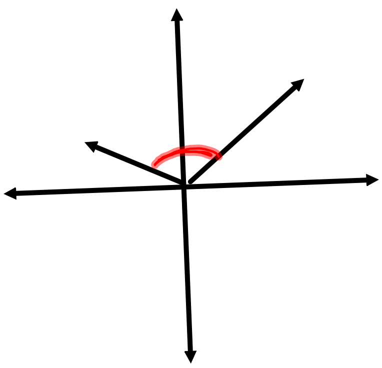


If θ is the angle between the 2 vectors u and v, then

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

Example 3. P672

Find the angle θ between the vectors $\langle -3, 1 \rangle$ & $\langle 5, 2 \rangle$



$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

$$\frac{-13}{\sqrt{10} \cdot \sqrt{29}} = \frac{-13}{\sqrt{290}}$$

$$\theta \approx 139.8$$

$$u \cdot v = -3 \times 5 + 1 \times 2 = -13$$

$$\|u\| = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$$

$$\|v\| = \sqrt{5^2 + 2} = \sqrt{29}$$

Find the angle θ between the vectors $\langle -2, 6 \rangle$ & $\langle 4, -1 \rangle$

$\theta = 122.47^\circ$

Shwarz inequality

$$|u \cdot v| \leq \|u\| \|v\|$$

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

Orthogonal ($\perp$) Vectors.

u and v are $\perp$ if

$u \cdot v = 0$

Example 4. P673.

a) $u = \langle 2, -6 \rangle, v = \langle 9, 3 \rangle$

Yes, $2(9) + (-6(3))$

$18 + (-18) = 0$

b) $\frac{1}{2}i + 5j$ and $10i - j$

$\frac{1}{2} \cdot 10 + 5 \cdot -1$

$5 + -5 = 0 \quad \perp$

Exercise (10.6A) PG79

19 → 21

$\boxed{21}$ $\langle 9, -6 \rangle, \langle -6, 4 \rangle$

$-54 - 24 \neq 0$

$\frac{-6}{9} = \frac{-2}{3}, \frac{4}{-6} = \frac{-2}{3}$

Parallel.

25, 26

27 $u = i - j \quad , v = ki + \sqrt{2}\, j$

$a = \langle 1, -1 \rangle , v = \langle k, \sqrt{2} \rangle$

$u \cdot v = 0$

$k - \sqrt{2} = 0 + \sqrt{2}$

$\boxed{k = \sqrt{2}}$

14, 17

Revision

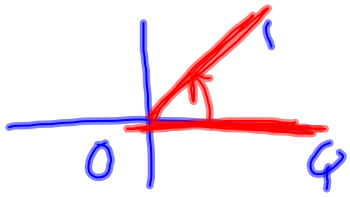
Tues, 24/2/15

Collection of Hw

P668

36

$OP = (20\cos(28) + 20\sin(28)$



$OQ = (30\cos(0) + 30\sin(0)$

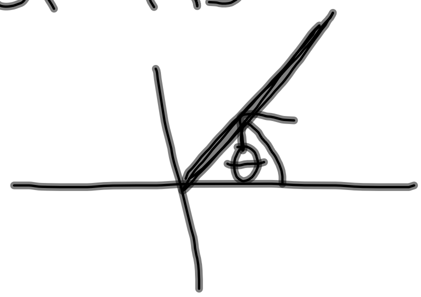
$OR = 47.7i + 9.4j$

$\|OR\| = \sqrt{(47.7^2) + (9.4)^2} = \boxed{48.6}$

$$\langle a, b \rangle$$

$$\downarrow$$

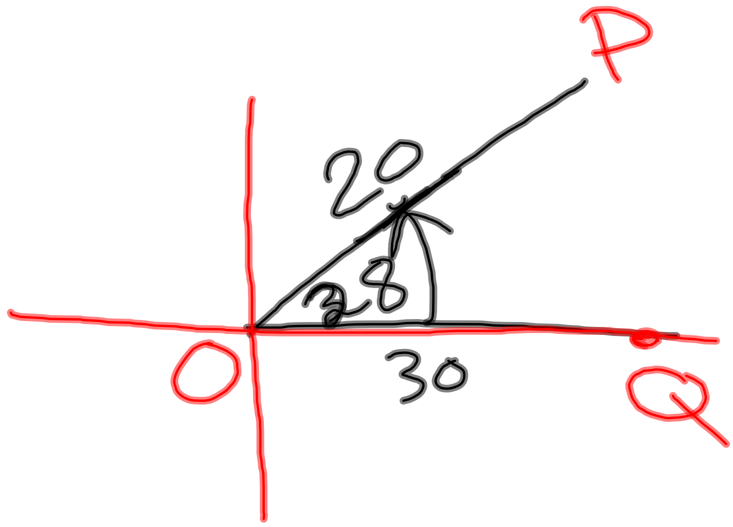
$$\sqrt{a^2+b^2}$$



$$\langle \overset{a}{\|v\|\cos\theta}, \|v\|\sin\theta \rangle$$

$$\langle \cdot , \cdot \rangle$$

36



$$\overrightarrow{OP} = 20\cos 28\, i + 20\sin 28\, j$$

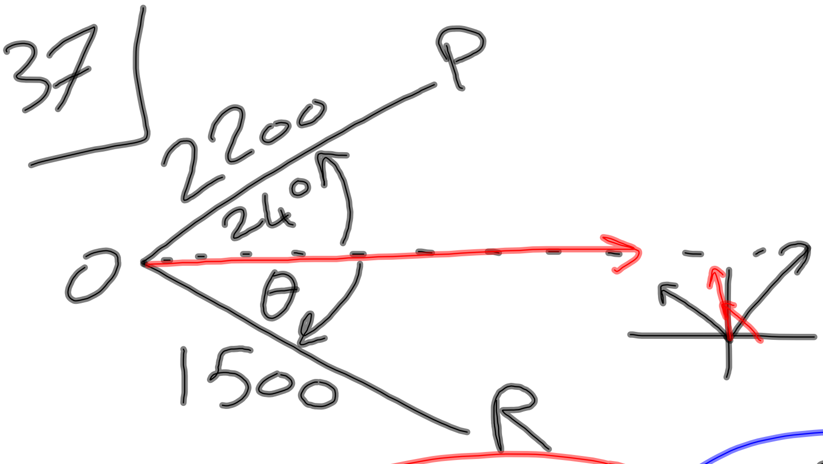
$$\langle 20\cos 28, 20\sin 28 \rangle$$

$$\overrightarrow{OQ} = \langle 30\cos 0, 30\sin 0 \rangle$$

$$\overrightarrow{OR} = \langle 47.7, 9.4 \rangle$$

$$\|\overrightarrow{OR}\| = \sqrt{47.7^2 + 9.4^2} = 48.6$$

37



$$\overrightarrow{OP} = \langle 2200\cos 24, 2200\sin 24 \rangle$$

$$\overrightarrow{OR} = \langle 1500\cos\theta, 1500\sin\theta \rangle$$

b

a

$$R = \langle 2009.8 + 1500\cos\theta, 894.8 + 1500\sin\theta \rangle$$

$$\tan 0 = \frac{b}{a} = 0$$

$$\frac{b}{a} = 0$$

$$\frac{894.8 + 1500\sin\theta}{2009.8 + 1500\cos\theta} = \frac{0}{1}$$

$$894.8 + 1500\sin\theta = 0$$

$$\frac{\cancel{1500}\sin\theta}{\cancel{1500}} = \frac{-894.8}{1500}$$

$$\sin\theta = \frac{-894.8}{1500}$$

$$\theta = -36.6°$$

$$R = \sqrt{\overset{a}{(2009.8+1500\cos\theta)} \quad \overset{b}{(894.8+1500\sin\theta)}}$$

$$\tan\theta = \frac{b}{a}$$

$$\frac{0}{1} = \frac{894.8+1500\sin\theta}{2009.8+1500\cos\theta}$$

$$0 = 894.8 + 1500\sin\theta$$

$$\frac{-894.8}{1500} = \frac{\cancel{1500}\sin\theta}{\cancel{1500}}$$

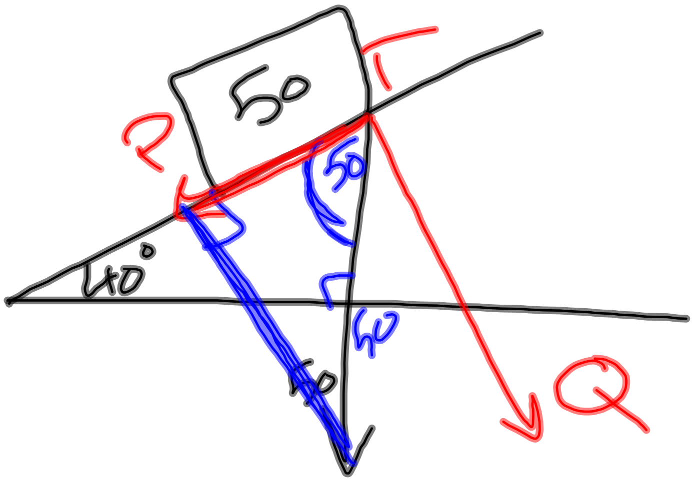
$$\sin\theta$$

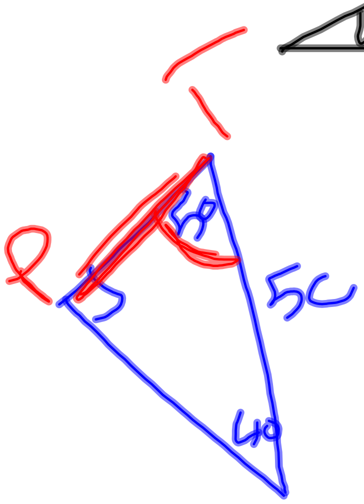
$$b = 0$$

zero

$$\frac{0}{a} = 0$$

$\boxed{38}$



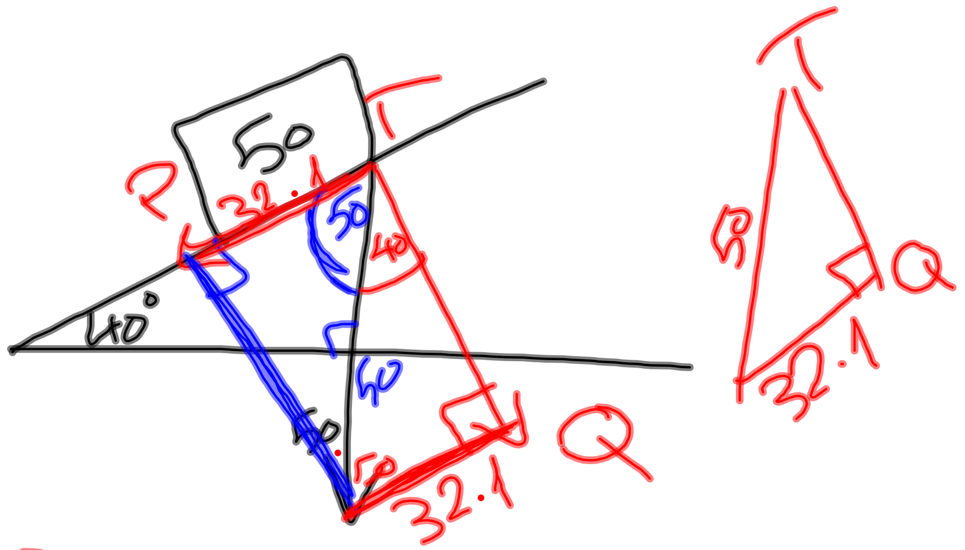


$$\cos 50 = \frac{\|\overrightarrow{TP}\|}{50}$$

$$\|\overrightarrow{TP}\| \doteq 32.1$$

OR

$$\sin 40 = \frac{\|\overrightarrow{TP}\|}{50} = 32.1$$



$$\|\overrightarrow{TQ}\| = \sqrt{50^2 - 32.1^2} = 38.3$$

$$\tan 40 = \frac{32.1}{\|TP\|}$$

Collection of H.w

P667

19) $||v|| = 3, \theta^\circ = 310^\circ$

$\langle a, b \rangle$

$\langle ||v|| \cos\theta, ||v|| \sin\theta \rangle$

$3 \cos 310, 3 \sin 310$

$\langle 1.928, -2.298 \rangle$

21] $v = \langle \overset{a}{5}, \overset{b}{5\sqrt{3}} \rangle$

$$\sqrt{5^2 + 5\sqrt{3}^2} = \boxed{10}$$

$$\tan\theta = \frac{5\sqrt{3}}{5}$$

$$\theta = 60^\circ$$

H.W

P 67 9 to 9, 10, 18, 22, 23, 24,
28

Correction of Hw

Mon, 2/3/15

P667

11) $u = i - 2j,\ v = 3i + j,\ w = -4i + j$

$3(u - 2v) - 6w$

$3\left(i - 2j - 2(3i + j)\right) - 6(-4i + j)$

$3\left(i - 2j - 6i - 2j\right) + 24i - 6j$

$3(-5i - 4j) + 24i - 6j$

$-15i - 12j + 24i - 6j$

$\boxed{9i - 18j}$

18) $||v|| = \frac{1}{2}, \theta = 250°$

$$\left\langle \frac{1}{2}\cos 250, \frac{1}{2}\sin 250 \right\rangle$$

$$\langle -0.17, -0.47 \rangle$$

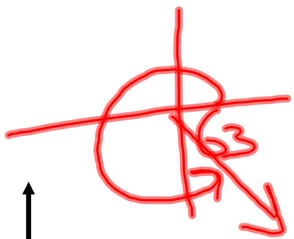
$$\langle \overset{a}{||u||\cos\theta}, \overset{b}{||u||\sin\theta} \rangle$$

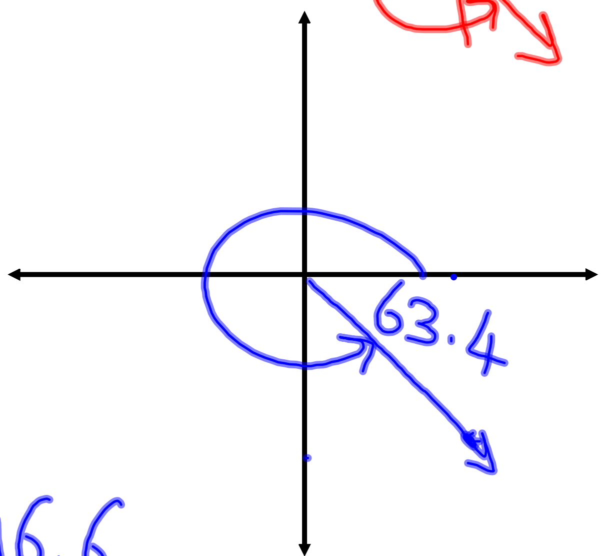
25) $V = 4i - 8j \quad , \langle 4, -8 \rangle$

$$\|V\| = \sqrt{4^2 + 8^2} = 4\sqrt{5}$$

$$\tan\theta = \frac{-8}{4} = -2$$

$$\theta = -63.4$$





direction angle =

$360 - 63.4 = 296.6$

26) $-2i+8j$

$\|v\| = \sqrt{2^2+8^2} = 8.24$

$\tan\theta = \frac{8}{-2} = -4$

$\theta = -75.9$

Direction angle $= 180 - 75.9 = 104.1$

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31) $-3i-9j$

$$\text{unit vector} = \frac{v}{\|v\|}, \quad \|v\| = \sqrt{3^2+9^2} = 3\sqrt{10}$$

$$v \cdot \frac{1}{\|v\|}$$

$$\text{unit vector} = \frac{\langle -3, -9 \rangle}{3\sqrt{10}}$$

$$\left\langle \frac{-\cancel{3}}{\cancel{3}\sqrt{10}}, \frac{-\cancel{9}}{\cancel{3}\sqrt{10}} \right\rangle = \left\langle \frac{-1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle$$

$$= \left\langle -\frac{\sqrt{10}}{10}, \frac{-3\sqrt{10}}{10} \right\rangle$$

34)

$$u = \langle 12\cos 130, 12\sin 130 \rangle$$

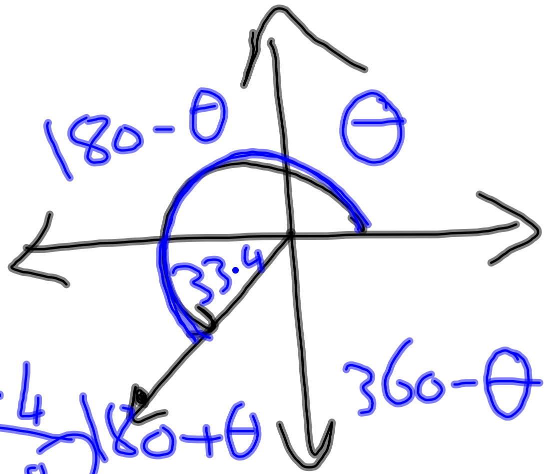
$$v = \langle 20\cos 250, 20\sin 250 \rangle$$

$$u+v = \langle -14.5, -9.6 \rangle$$

$$\|u+v\| = \sqrt{14.5^2+9.6^2} = 17.4$$

$$\tan\theta = \frac{-9.6}{-14.5}$$

$$\theta = 33.41$$



Direction $= 180+33.4$
$= 213.4°$

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9) $u = \langle 2, 5 \rangle$, $v = \langle -4, 3 \rangle$, $w = \langle 2, -1 \rangle$

$(u + v) \cdot (v + w)$

$\langle -2, 8 \rangle \cdot \langle -2, 2 \rangle =$

$4 + 16 = \boxed{20}$

18

$u = 3i - 5j$, $v = -2i + 3j$

$\|u\| = \sqrt{3^2 + 5^2} = \sqrt{34}$

$\|v\| = \sqrt{2^2 + 3^2} = \sqrt{13}$

$u \cdot v = -6 - 15 = \boxed{-21}$

$$\cos\theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$= \frac{-21}{\sqrt{34} \cdot \sqrt{13}}$$

$\theta = 177.3$

Tell if the vectors are Parallel.

$u = \langle 1, 3 \rangle, v = \langle 0, 0 \rangle$

Parallel

$k = 0$

$v = k \cdot u$

$\langle 0, 0 \rangle = 0 \langle 1, 3 \rangle$

$u = \langle 4, 3 \rangle, v = \langle -1, 4 \rangle$

$-4 + 12 = \boxed{8}$

Neither.

$6i - 4j \quad , 2i + 3j$

$12 - 12 = 0 \quad \perp$

$-i + 2j \quad , 2i - 4j$

$\langle -1, 2 \rangle \quad , \langle 2, -4 \rangle$

$-\cancel{2 - 8 = \boxed{-10}}$

$\frac{-2}{-2}$