12.3 Matrix Operations

Tues, 21/4/15

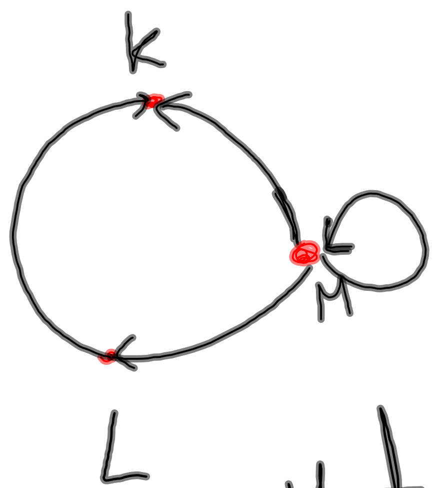
30

$$
\begin{array}{r|cc} & \text{Nuts} & \text{fruit} \\ \hline \text{fat} & & \\ \text{Protein} & \multicolumn{2}{c}{\left[\begin{array}{cc} 1g & 52g \\ 3g & 20g \\ 65g & 21g \end{array}\right]} \\ \text{carbs} & & \end{array}
\qquad
\begin{array}{r|cc} & A & B \\ \hline \text{Nuts} & \multicolumn{2}{c}{\left[\begin{array}{cc} 30\% & 45\% \\ 70\% & 55\% \end{array}\right]} \\ \text{fruit} & & \end{array}
$$

$$
\begin{array}{r|cc} & A & B \\ \hline \text{fats} & & \\ \text{prot.} & \multicolumn{2}{c}{\left[\begin{array}{cc} 36.7 & 29.05 \\ 14.9 & 12.35 \\ 34.2 & 40.8 \end{array}\right]} \\ \text{carbs} & & \end{array}
$$

- Serving A contains 36.7g of fats, 14.9 g. of Proteins, and 34.2g of Carbs
- Serving B contains 29.5g of fats, 12.35 g of Proteins, and 40.8 g. of Carbs.

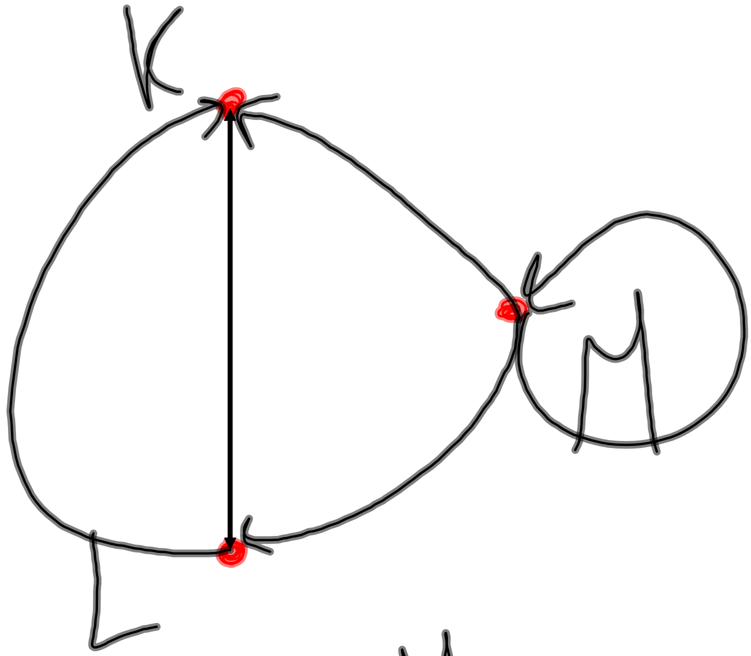
Directed Network



Adjacency Matrix

	K	L	M
K	0	0	0
L	1	0	0
M	1		

1- Stage Path.



K L M

K

L

M

Square Matrix.

Is a matrix with dimensions $\boxed{n \times n}$ where the no of rows equals the no of columns.

The Determinant of the Square matrix is the difference between the diagonals starting from top left.

A 2x2

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\text{Det } A = 1 \cdot 5 - 4 \cdot 3 = \boxed{-7}$$

B 3x3

$$\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & -6 \\ 5 & 0 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & -6 \\ 5 & 0 & 7 \end{bmatrix}$$

$$\left[(1 \cdot 1 \cdot 7 + 4 \cdot -6 \cdot 5 + 2 \cdot 3 \cdot 0) - (5 \cdot 1 \cdot 2 + 0 \cdot -6 \cdot 1 + 7 \cdot 3 \cdot 4) \right] =$$

12.4 Matrix Methods for Square Systems

Every square matrix $\boxed{n \times n}$ has an identity matrix I_n

$$I_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* for Any $n \times n$ Matrix A

$$A I_n = I_n A = A$$

Example 2. P815

$$C = \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}$$

$$I_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_2 C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & \boxed{2} \\ 5 & \boxed{7} \end{bmatrix} =$$

$$1 \cdot 2 + 0 \cdot 7$$
$$2 + 0$$
$$= 2$$

$$= \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}$$

Example.

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$

Find AB, BA,

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

* Matrix B is the inverse Matrix of matrix A.

* Inverse Matrix.

Is the matrix when multiplied by a given matrix the Product is In → Identity matrix

Example 4. P816

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$$

$$A A^{-1} = I_2$$

$$\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x & m \\ y & n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2x + 6y = 1 \qquad 2m + 6n = 0$$
$$x + 4y = 0 \qquad m + 4n = 1$$

$$x = 2 \qquad y = -\tfrac{1}{2} \qquad m = -3 \qquad n = 1$$

$$A^{-1} \begin{bmatrix} 2 & -3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

* Solving linear Square Systems using matrices.

Example 5.P817.

$$x + y = 2$$
$$2x - 9y = 15$$

Coefficient Matrix Variable Matrix Constants Matrix

$$A \begin{bmatrix} 1 & 1 \\ 2 & -9 \end{bmatrix} \quad X \begin{bmatrix} x \\ y \end{bmatrix} \quad B \begin{bmatrix} 2 \\ 15 \end{bmatrix}$$

$$A^{-1} B = X$$

$$X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$x = 3, y = -1$

$(3, -1)$

Example 6. P818

$x + y + z = 2$

$2x + 3y = 5$

$x + 2y + z = -1$

$A \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad B \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$

$x \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad A^{-1}B = \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$

$\underline{3} \times \boxed{3 \quad 3} \times \underline{1}$

Exercise 12.4 P819

4, 8, 16, $\underline{\underline{17}}$

16) $\begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$

17) No inverse

* If the determinant of a square matrix = 0, then the matrix has no inverse
→ Singular matrix

H.w

P819 № 7, 18, 22, 24

Mon, 27/4/15

12-3 Matrix operations

Directed Network.

Any directed network
Could be represented by
a matrix called. adjacency matrix

Mon, 27/4/15

12-4 Matrix Methods for Square Systems

$x+y+z=2$

$2x+3y \quad =5$

$x+2y+z=-1$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix}} \quad \underset{B}{\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}} \quad \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}$$

$3\times\boxed{3 \quad 3}\times 1$ ✓

$$A^{-1}\cdot B = \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix} \begin{array}{l} \to x \\ \to y \\ \to z \end{array}$$

Example 7. P818

$$(\overset{x}{1}, \overset{y}{-4}), (\overset{x}{-1}, \overset{y}{-10}), (\overset{x}{4}, \overset{y}{-25})$$

$$y = ax^2 + bx + c$$

$$-4 = a + b + c$$

$$-10 = a - b + c$$

$$-25 = 16a + 4b + c$$

$$\overset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \end{bmatrix}}^{-1} \times \overset{B}{\begin{bmatrix} -4 \\ -10 \\ -25 \end{bmatrix}} =$$

Exercise (12-4). P820.

40

41 $y = ax^3 + bx^2 + cx + d$

43 $y = ax^4 + bx^3 + cx^2 + dx + e$

12.5 Nonlinear Systems

Example 1. P821

$$x^2 - \cancel{y^2} = 5$$
$$x^2 + \cancel{y^2} = 13 \quad +$$

$$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{18}{2}$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

$$\boxed{x^2 + y^2 = 13}$$
$$9 + y^2 = 13$$
$$y^2 = 4$$
$$y = \pm 2$$

Example 2.

$$2x^2 - y^2 = 1$$

$$x + y = 12 - y$$

$$x = 12 - y$$

$$2(12 - y)^2 - y^2 = 1$$

$$2(144 - 24y + y^2) - y^2 = 1$$

$$288 - 48y + 2y^2 - y^2 - 1 = 0$$

$$y^2 - 48y + 287 = 0$$

$\boxed{y = 41}$ or $\boxed{y = 7}$

$$x = 12 - y$$

$x = 12 - 41$ or $x = 12 - 7$

$\boxed{x = -29}$ $\boxed{x = 5}$

Example 4 . P823

$$2x^2 + 3y^2 = 30$$

$$2x^2 - xy - y^2 = 8$$

$$\frac{\cancel{3}}{\cancel{3}}y^2 = \frac{30-2x^2}{3}$$

$$y = \pm\sqrt{\frac{30-2x^2}{3}}$$

$$2x^2 - xy - y^2 = 8$$

Tues, 28/4/15

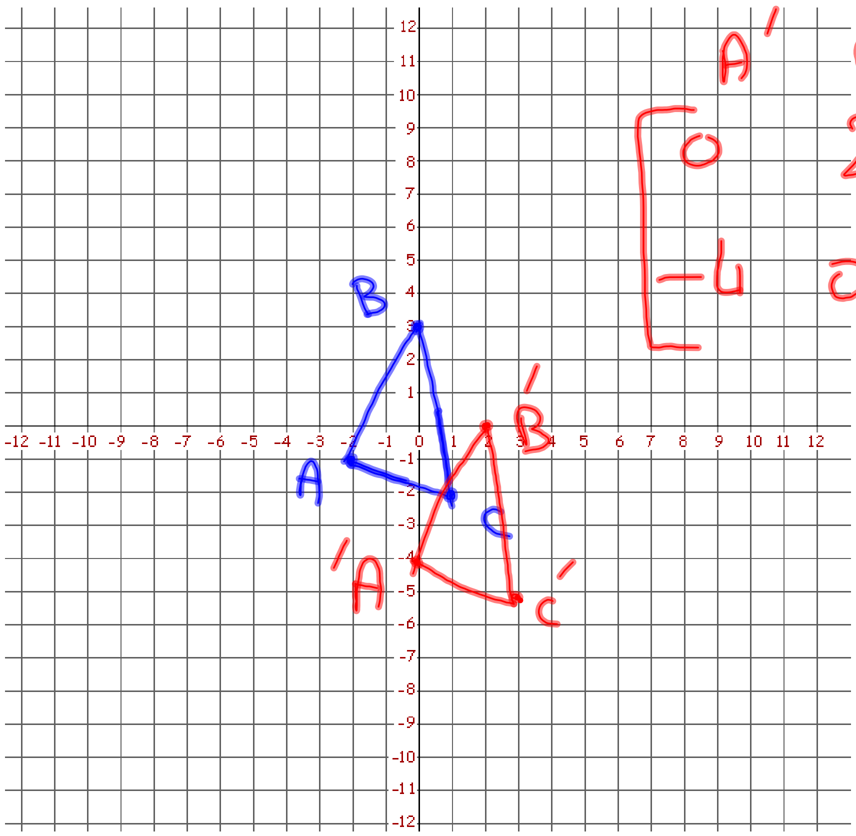
Using Matrices to Transform Geometric Figures

Translate $\triangle ABC$, with $A(-2,-1)$, $B(0,3)$, $C(1,-2)$ 2 units right and 3 units down.

Translation Matrix.

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} x \rightarrow \\ y \rightarrow \end{array} & \begin{bmatrix} -2 & 0 & 1 \\ -1 & 3 & -2 \end{bmatrix} \end{array} + \begin{array}{c} x \rightarrow \\ y \rightarrow \end{array} \begin{bmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}$$

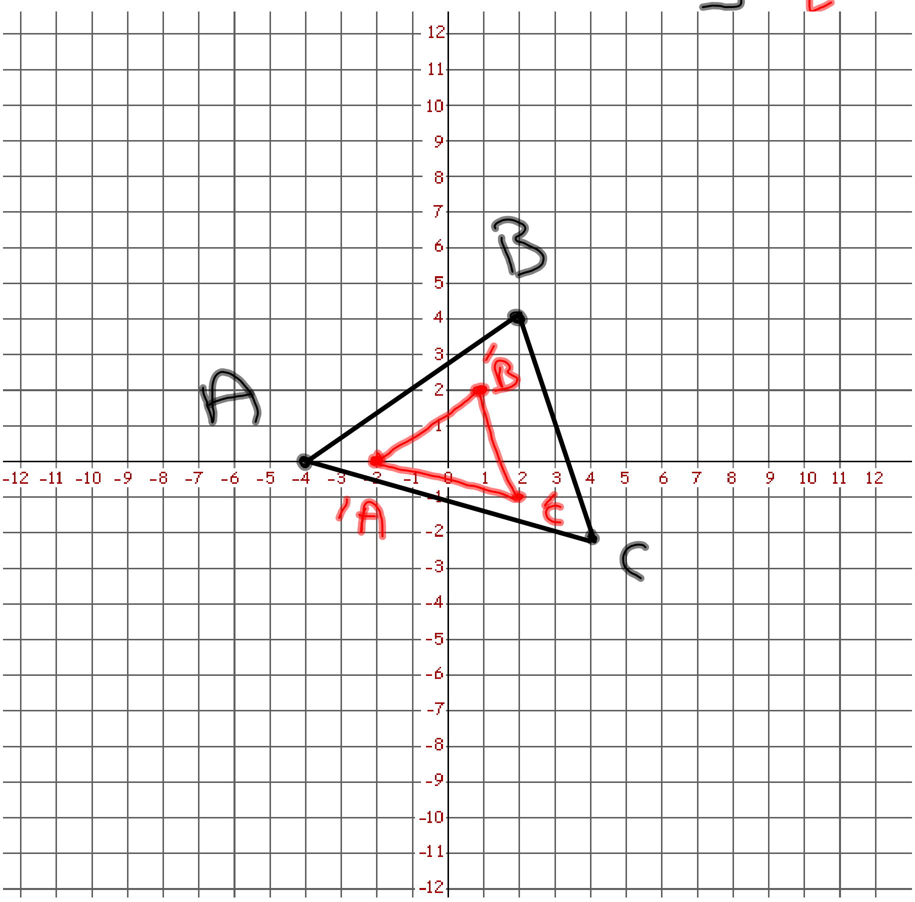
$$\begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & 2 & 3 \\ -4 & 0 & -5 \end{bmatrix} \end{array}$$



Dilation

Reduce $\triangle ABC$ with $A(-4,0)$ $B(2,4)$, and $(4,-2)$ by a factor of $\frac{1}{2}$

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} x \\ y \end{array} & \frac{1}{2}\begin{bmatrix} -4 & 2 & 4 \\ 0 & 4 & -2 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} -2 & 1 & 2 \\ 0 & 2 & -1 \end{bmatrix} \end{array}$$



Reflection matrix across x-axis

To reflect a figure across the x-axis multiply the coordinates matrix by

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Reflection matrix across y-axis

To reflect a figure across the y-axis multiply the coordinates matrix by

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad 2 \times \boxed{2 \quad 2} \times \cdot = -$$

Excercise.

In ΔJKL, $J(3,4)$, $K(4,2)$,

and L is $(1,-2)$

a) Reflect ΔJKL across y-axis

b) " " " " " " " x-axis

c) Graph both images

a) across x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} J & K & L \end{array} \\ \begin{bmatrix} 3 & 4 & 1 \\ 4 & 2 & -2 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} J' & K' & L' \end{array} \\ \begin{bmatrix} 3 & 4 & 1 \\ -4 & -2 & 2 \end{bmatrix} \end{array}$$

b] across y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} J & K & L \end{array} \\ \begin{bmatrix} 3 & 4 & 1 \\ 4 & 2 & -2 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} J'' & K'' & L'' \end{array} \\ \begin{bmatrix} -3 & -4 & -1 \\ 4 & 2 & -2 \end{bmatrix} \end{array}$$

Rotation Matrices.

1) To rotate a figure 90° Counter Clock wise multiply by $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2) To rotate a figure 90° clockwise multiply by

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

3] To rotate a figure 180°

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

1) Rotate the Polygon ABCD

Such $A(0,0), B(4,2), C(2,-5)$

$D(-1,-3)$ by.

a) An angle 90° counter-clock wise

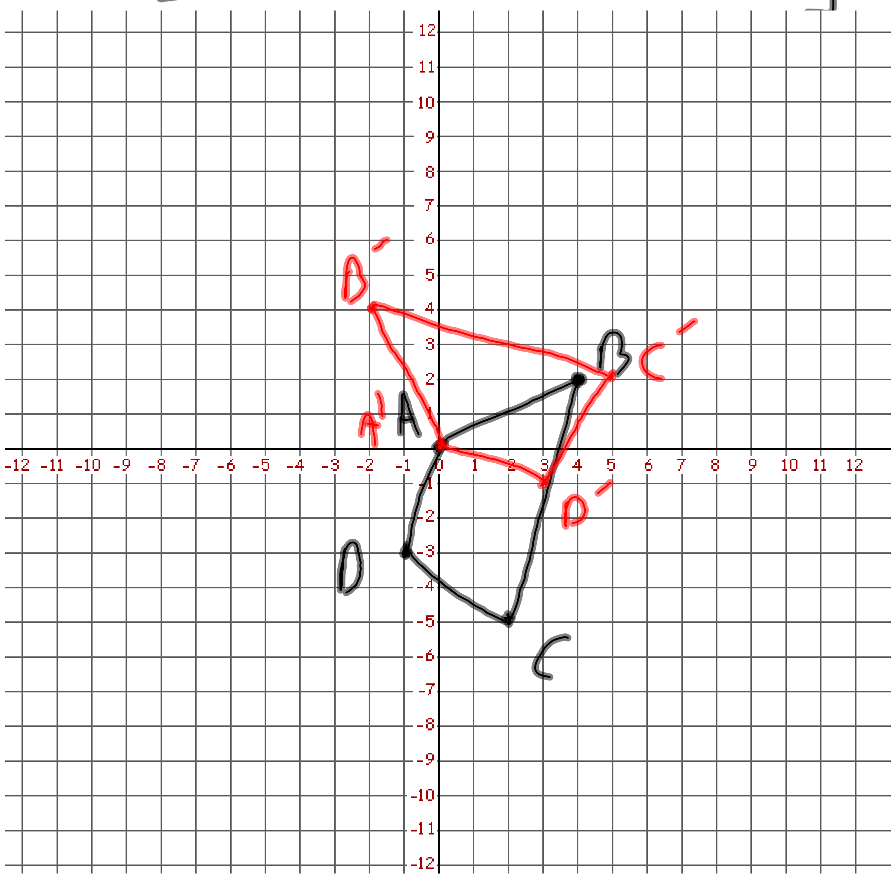
b) An angle 90° Clock wise

c) An angle 180°

$$\begin{array}{c}\begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 2 & -1 \\ 0 & 2 & -5 & -3 \end{bmatrix}\end{array}$$

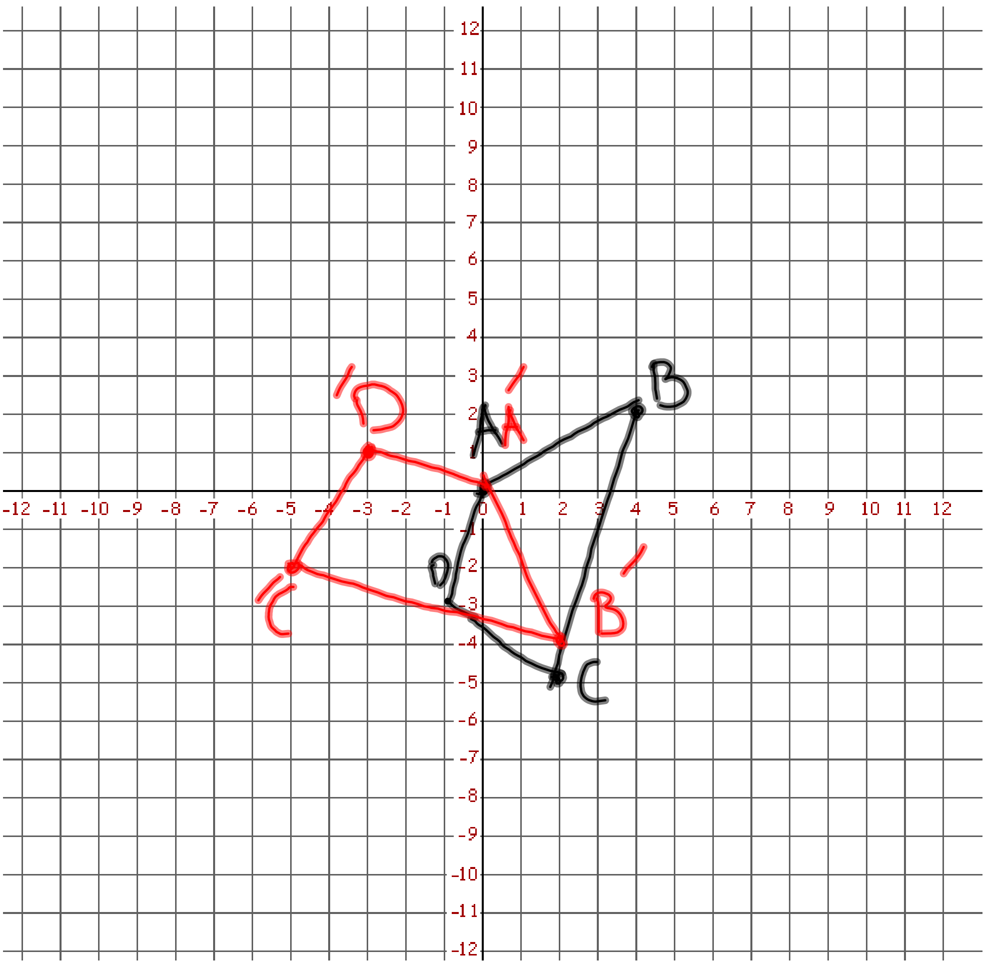
a) $$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{array}{c}\begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 2 & -1 \\ 0 & 2 & -5 & -3 \end{bmatrix}\end{array}$$

$$= \begin{array}{c}\begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & -2 & 5 & 3 \\ 0 & 4 & 2 & -1 \end{bmatrix}\end{array}$$



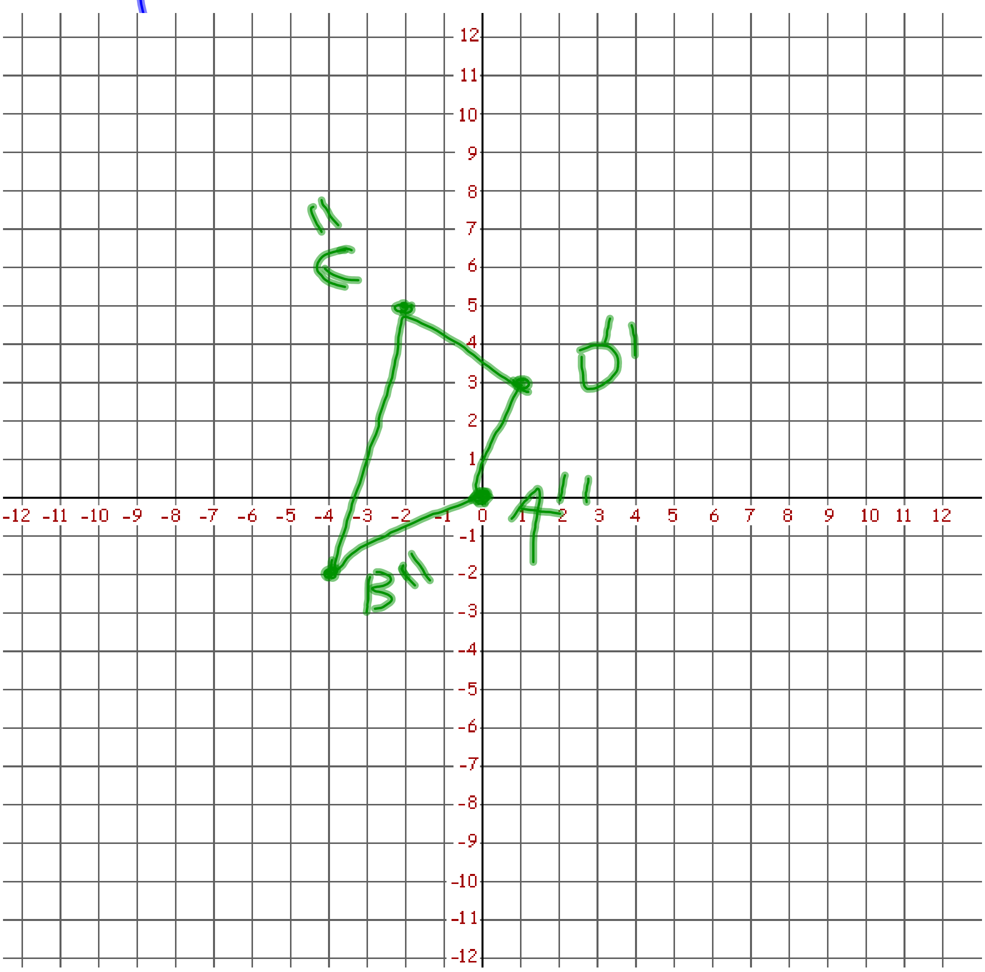
b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 & -1 \\ 0 & 2 & -5 & -3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 & -5 & -3 \\ 0 & -4 & -2 & 1 \end{bmatrix}$



c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 & -1 \\ 0 & 2 & -5 & -3 \end{bmatrix}$

$$\begin{bmatrix} 0 & -4 & -2 & 1 \\ 0 & -2 & 5 & 3 \end{bmatrix}$$



Exercise.

1) Enlarge Polygon PQRS where P $(-2,4)$, Q$(3,1)$, R$(1,-4)$, S$(-2,-2)$ by a factor of 2

b] Translate PQRS by the matrix $\begin{bmatrix} -1 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$

c] Rotate the figure by $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and describe the angle of rotation.

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