Tues, 5/5/15

13.3 Basic Probability

*Probability is the measure of how likely an event is to happen

*Experiment. is any Process the generates one or more Possible outcomes.

*Event. is outcome or set of outcomes in the sample space.

$P(\text{boy}) = \frac{5}{16}$

$P(\text{even no}) = \frac{3}{6} = \frac{1}{2}$

* Probability Distribution.

Is to describeall the events in a function, with a domain of Sample Space and range in the interval [0,1]

a) Example 1. P865.

a) Sample space: red, yellow, blue, and green

red $\to$ 50, yellow $\to$ 10, blue $\to$ 30, green $\to$ 10

b)

color	Red	Yell.	blue	green
Prob	0.5	0.1	0.3	0.1

c) $P(\text{blue } \underline{\text{or}} \text{ green}) = 0.3 + 0.1$

$= 0.4$

+

* Mutually Exclusive Events.

- Are events that have no common outcomes.
- The 2 events can not happen at the same time.

Example.

When rolling a fair die, tell whether the following events are mutually exclusive events.

a) Getting an even no and an odd no

$E = \{2, 4, 6\}$, $O = \{1, 3, 5\}$ Mutually Exclusive

b) getting a no less than 4, and a multiple of 5

$\{1, 2, 3\}$, $\{5\}$ Mutually Exc.

c) getting an odd no and a multiple of 3

$\{1, 3, 5\}$

$\{3, 6\}$

Not mut. exc.

d) Getting a no greater than 2, and a factor of 8

$\{3, 4, 5, 6\}$, $\{1, 2, 4\}$

Not mut. exc.

* Independent Events.

Two events the occurance of one of them doesn't affect the Probability of the other.

* DePendent Events.

Are 2 events the occurence of one of them affects the Probability of the other.

Example. Tell if the following events are dependent or independent, and find the probability of each.

2 Cards are drawn from a card deck of 52.

a] Selecting 2 aces when the 1st card is replaced.

Independent. 1st draw and 2nd draw

$$P(2\text{aces})\ \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

b) Selecting 2 aces when 1st card is not replaced.

Dependent

$$P(2\text{aces}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$$

c) Selecting a face card without replacing it, then selecting a 7.

dependent event

$$\frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$$

d] Selecting 2 hearts, and the 1st card is replaced.

Independent $\frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$

e] Selecting a queen without replacing it, then selecting a king

Dependendent.

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$$

* If A and B are independ events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

* If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B/A)$

Conciderinġ that A has happened.

* Complement of an event.

Is the set of all outcomes that are not contained in the event.

Complement of an event $\rightarrow P'$

$P' = 1 - P$

Exercise. 13.3 P 8-12

5)

Col.	black	red	white
Prob	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$P(\text{white}) = 1 - \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{6}$$

6) $P(3\text{ blacks}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

7) $P(3\text{ white}) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$

8) $P(b, w, r) = \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{36}$

$P(red, w, b) = \frac{1}{3} \cdot \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{36}$

$\frac{1}{36}$ $\frac{1}{36}$ $\frac{1}{36}$ $\frac{1}{36}$ $\frac{1}{36}$ $\frac{1}{36}$

(rwb), (rbw), (wrb), (wbr), (brw), (bwr)

$$\frac{1}{36} \cdot 6 = \frac{1}{6}$$

9)

10)

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13.3 Basic Probability

Mutually ex.

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Ind. or dep.

Inclusive events.

Are events that have one or more common outcomes.

$$P(A \text{ or } B) = P(A) + P(B) - P(\underbrace{A \text{ and } B}_{\text{togather}})$$

Example.

Find each Probability when Rolling a die.

1) Rolling an even no or a Prime no.

$$P(E \text{ or } P) = \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$$

2] Rolling a 5 or an odd no.

$$P(5 \text{ or } O) = \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = \frac{1}{2}$$

3] Rolling an odd no or a no greater than 2

$$P(O \text{ or } >2) = \frac{1}{2} + \frac{4}{6} - \frac{2}{6} = \boxed{\frac{5}{6}}$$

Exercise

of 3510 drivers surveyed, 1950 were males, and 103 were color-blind. only 6 of the color-blind drivers were females. what is the probability that a driver is chosen at random is a male or color-blind?

$$P(M \text{ or } CB) = P(M) + P(CB) - P(M, CB)$$

$$\frac{1950}{3510} + \frac{103}{3510} - \frac{97}{3510} =$$

103-6 Fem

$$\frac{1950+103-97}{3510} = \frac{326}{585}$$

$$\boxed{0.557}$$

Exercise.

Of 160 beauty SPa Customers 96 had a hair Styling and 61 had a manicure. There were 28 Customers who had only a manicure. What is the Probability that a customer had a hair styling or a manicure?

* Random Variable.

Is a function that assigns a no to each outcome.

* Expected value.

Is the average value of the outcomes.

Exercise 13.3. P872, 873.

$\boxed{11}$ $\{0, 1, 2\}$

0 → No blue, 1 → one blue only, 2 → both blue

~~No blue~~ 0	~~only 1 blue~~ 1	~~both blue~~ 2
$\frac{25}{64}$	$\frac{15}{32}$	$\frac{9}{64}$

$\boxed{12}$ $P(\underset{2}{\text{both blue}}) = \frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$

$\boxed{13}$ $P(\underset{0}{\text{Neither blue}}) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$

$\boxed{14}$ 0

$\boxed{15}$ $1 - \left(\frac{25+9}{64}\right) = \frac{30}{64} = \frac{15}{32}$

or

$P(\text{only one blue}) = \frac{3}{8} \cdot \frac{5}{8} + \frac{5}{8} \cdot \frac{3}{8} = \frac{15}{32}$

or

1st blue, 2nd not blue

1st not blue, 2nd blue.

$\boxed{16}$ Expected value =

$0 \cdot \frac{25}{64} + 1 \cdot \frac{15}{32} + 2 \cdot \frac{9}{64} = \boxed{\frac{3}{4}}$

21) P(at least one is absent)

$P(1) + P(2) + P(3) + P(4) + P(5) =$

$0.33 + 0.07 + 0.01 + 0 + 0 = \boxed{0.41}$

OR

P(at least one is absent) =

$1 - P(0 \text{ absent})$

$1 - 0.59 = \boxed{0.41}$

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13.4 Determining Probabilities

Fundamental counting Principle.

$$\underset{1}{\overset{26}{\_}}\ \overset{26}{\_}\ \overset{26}{\_}\ \overset{26}{\_} = 26^4 = \boxed{456976}$$

Ordering the elements of one group

- Factorial !
  - $\downarrow$ all.
  - $10!$
- Permutation
  - $\downarrow$ some
  - $10P3$
  - nPr
  - $$\frac{n!}{(n-r)!}$$
- combination
  - $\downarrow$ some
  - $10C3 = 120$
  - nCr
  - $$\frac{n!}{r!\,(n-r)!}$$

$$n\,!\,r = \frac{\quad}{(n-r)!} = \frac{10.}{\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{\ldots}}$$

B.P88318 → 26.

18) $5 \cdot 2 \cdot 3 = \boxed{30}$

19) T.F M.C.Q

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 4 \cdot 4 \cdot 4 =$

$2^5 \cdot 4^3 = 2048$

digits lett.

20) $\cancel{10} \cdot \cancel{10} \cdot \cancel{10} \; \cancel{26} \cdot \cancel{26} \cdot \cancel{26}$

$$26^3 \cdot 10^3 = 17,576,000$$

21) $25 C_{10} = 3,268,760$

22) ${}_8P_3 = 336$

23) $9! = 362{,}880$

24) ${}_{700}C_3 = 56{,}921{,}900$

25) $\frac{1}{4!} = \boxed{\frac{1}{24}}$

26) $\frac{1}{{}_{50}P_6} = 8.74 \times 10^{-11}$

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13.4 A Binomial Experiments

A binomial experiment is an experiment that has only 2 possible outcomes, either success or failure.

* $P(r \text{ successful trials})$

$$nCr \cdot P^r \cdot q^{n-r}$$

$n \rightarrow$ no of trials

$r \rightarrow$ no of successful trials

$P \rightarrow$ Probability of success

$q \rightarrow$ Probability of failure $(1-P)$

Example 1. P884

In a basket ball contest, each contestant is allowed 3 free-throws. If a player has a chance 70% of making each throw. What is the probability of making exactly 2 out of 3 throws?

$$\boxed{nC_r \cdot p^r q^{n-r}}$$

$$3C_2 \cdot (0.7)^2 \cdot 0.3^1 = 0.441$$

Exercise

A baseball player has a probability 0.3 of getting a hit. What is the probability of getting exactly 1 hit of 4 times at bat?

$$4C_1(0.3)^1(0.7)^3 = 0.4116$$

Example 2. P886

$\$1000$ 1000

2 times or more

$P(\text{2 times or more}) =$

$P(2)+P(3)+P(4)+P(\text{5 or more})$

or

$1-(P(0)+P(1)) = 0.2642$

Example3. P887

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Correction of HW

2) $_5P_2 = 20$

OR

$\underline{5} \cdot \underline{4} = 20$

4] $5P3 = 60$

6] $11C_3 = 165$

8] $9P4 = 3024$

10) $6P3 = 120$

13 $9 \cdot 25 = 225$

14 $8C_2 = 28$

1 2 3 4 5 6 7 8

$\frac{1}{28}$

OR

$\frac{\cancel{2}^{1}}{\cancel{8}_{4}} \times \frac{1}{7} = \boxed{\frac{1}{28}}$

$\boxed{15}$ ${}_8P_3 = 336$

$\boxed{16}$ ${}_{12}P_3 = 1320$

$\boxed{17}$ ${}_5C_3 = 10$

18 $4 \times 3 = 12$

---

19 $9P2 = 72$

---

20 $5P2 = 20$

21) ${}_{10}P_4 = 5040$

Sample space

0123, 1234, 2345, 3456, 4567,

5678, 6789

$$\frac{14}{5040} = \boxed{\frac{1}{360}}$$

13.4 A Binomial Experiments

Expected Value for a binomial experiments

$= nP$

Probability of Success

no of trials